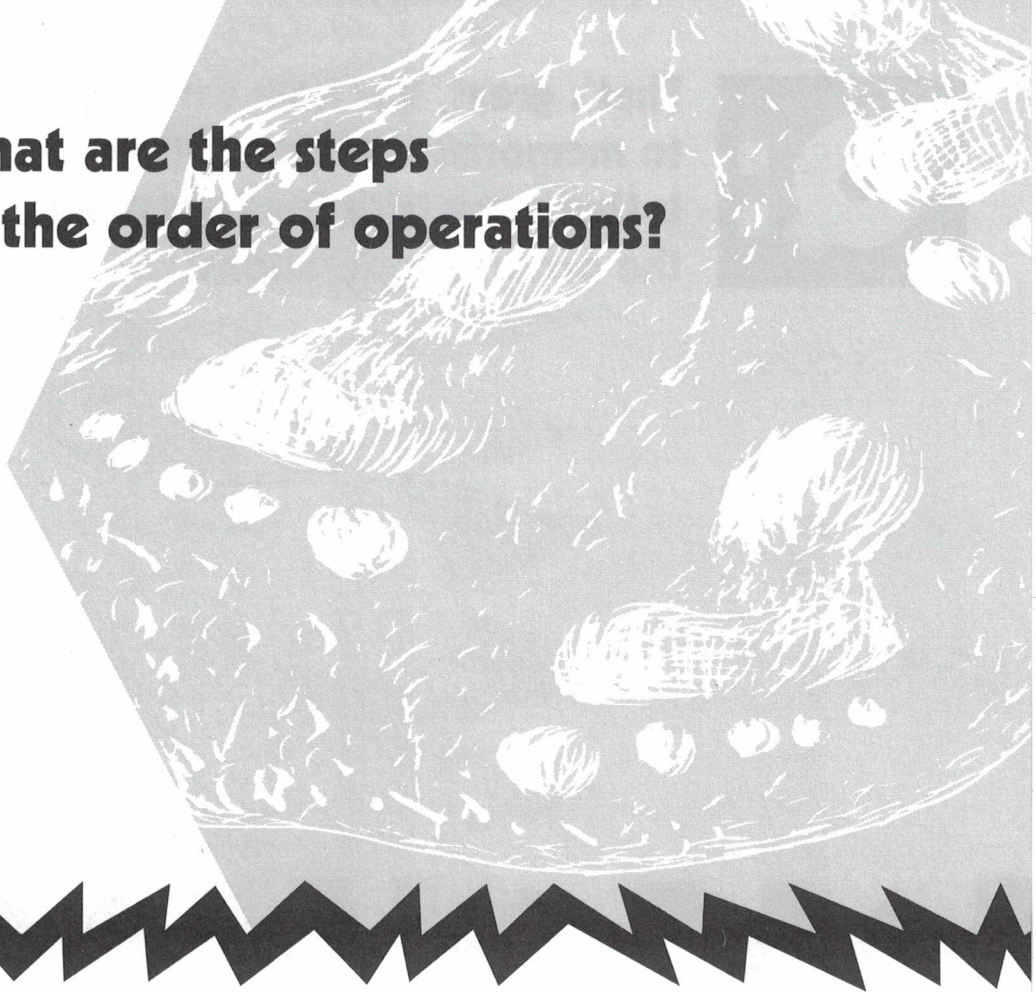


What are the steps in the order of operations?



Below you'll find the steps in the order of operations.

The sooner you memorize these, the easier it'll be for you to simplify expressions.

1st) Work out operations in **parentheses** or other enclosure marks (brackets, absolute value signs, etc.), working from the innermost enclosure marks to the outermost ones.

2nd) Raise terms to **exponents**.

3rd) Perform all **multiplication** and **division** operations, doing whichever operation comes first as you work from left to right.

4th) Use the **neighbor-sign rule**.

5th) **Group** positive numbers with positives and negatives with negatives.

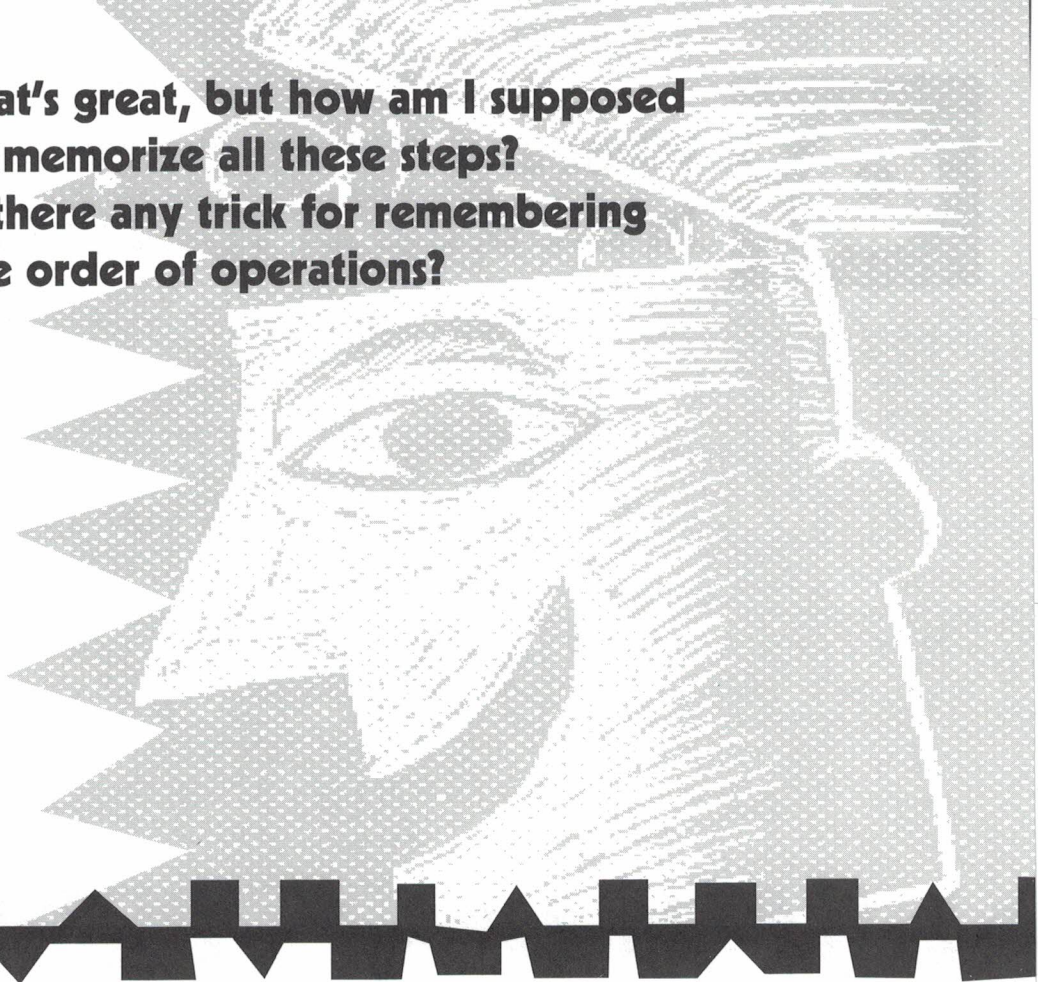
6th) Use the **same-sign rule**.

7th) Use the **mixed-sign rule** to get your answer.

Note: If you don't yet know how to raise numbers to exponents, just look at the first page of the Exponents section (p. 86), and you'll find out how.



That's great, but how am I supposed to memorize all these steps? Is there any trick for remembering the order of operations?



Yes, there's a memory trick. And if you've read through the section on Positive and Negative Numbers, you already know the last half of the trick. But now you get the whole sentence:

Please Eat More Dessert — Nate's Great Strawberry Mousse.

P stands for **P**arentheses.

E stands for **E**xponents.

M stands for **M**ultiplication.

D stands for **D**ivision.

N stands for **N**eighbor-sign rule.

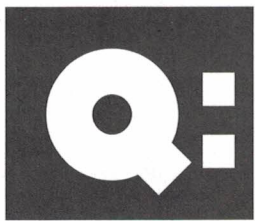
Gr stands for **G**rouping.

S stands for **S**ame-sign rule.

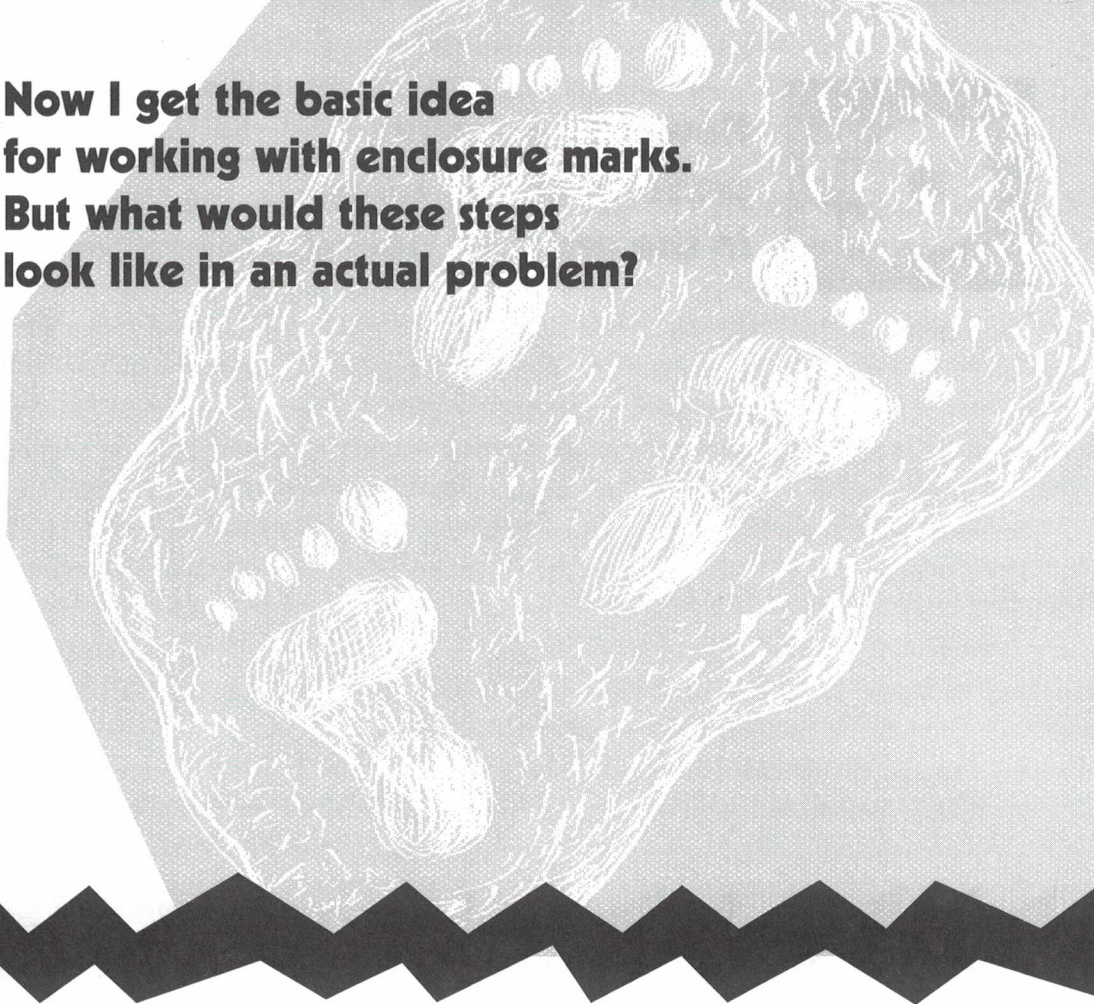
M stands for **M**ixed-sign rule.

For those of you who've never had the pleasure, mousse is a light, airy dessert. Some people describe mousse as "pudding with wings."





Now I get the basic idea for working with enclosure marks. But what would these steps look like in an actual problem?



Below you'll see just such an example. Simplify: $4 \cdot [18 - \{(12 \div 2) + 4\}]$

Steps

1st) Identify the innermost enclosure marks, compute the value and plug it in.

2nd) Identify next innermost enclosure marks, compute the value, plug it in.

3rd) Identify final enclosure marks, compute the value, plug it in. Then work out your answer.

Think it out

Innermost is $(12 \div 2)$
 $12 \div 2 = 6$

Next is $\{6 + 4\}$
 $6 + 4 = 10$

Final is $[18 - 10]$
 $18 - 10 = 8$

Work it out

$$4 \cdot [18 - \{(12 \div 2) + 4\}]$$

$$= 4 \cdot [18 - \{6 + 4\}]$$

$$4 \cdot [18 - \{6 + 4\}]$$

$$= 4 \cdot [18 - 10]$$

$$4 \cdot [18 - 10]$$

$$= 4 \cdot 8$$

$$= 32$$



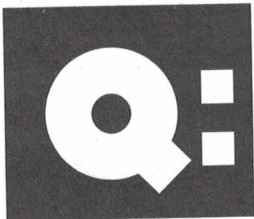
Now you try. Simplify using the inner-to-outer order for enclosure marks:

a) $3 \cdot \{24 - [2 + (5 \cdot 2)]\}$

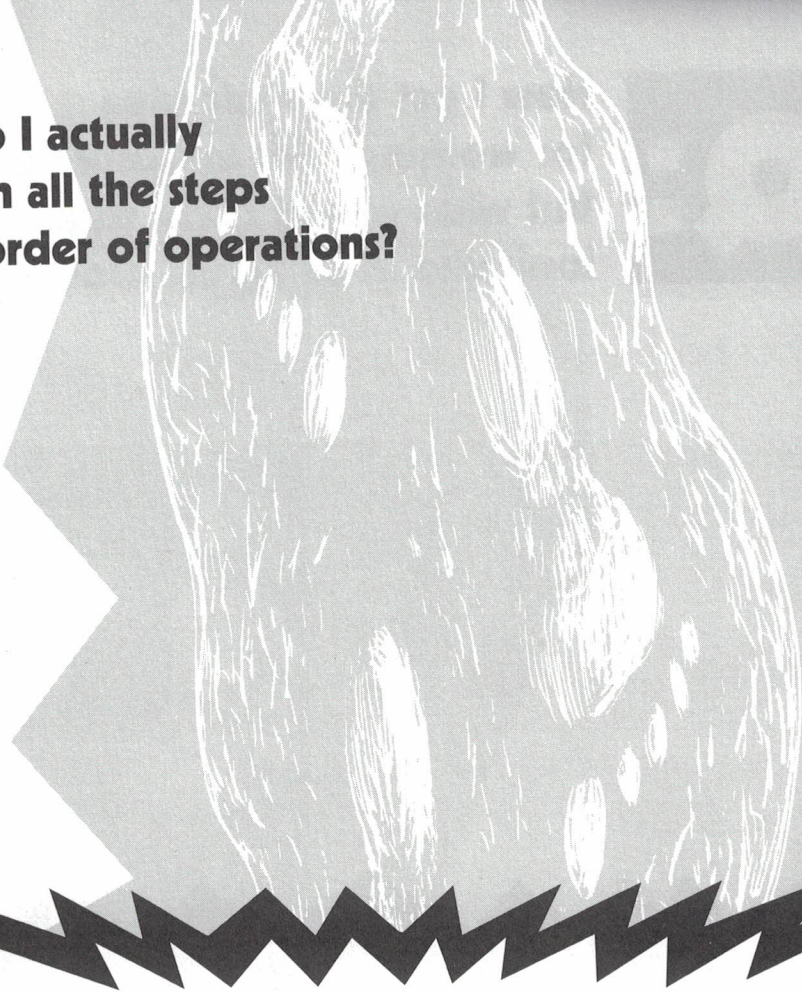
b) $6[4\{2 + (3 \cdot 1)\} + 2(8 - 3)]$

c) $7[(2 + 3) \cdot (2 - 3)]$

Answers:
 a) 36
 b) 180
 c) -35



How do I actually perform all the steps in the order of operations?



The following problem — **and yes, it's a doozy!** — shows how to use all the steps in the order of operations. Keep in mind that few problems would require you to use **all** these steps.

Simplify using the order of operations: $+ 2[8 - 4] - (+ 3) + 5^2 - 20 \div 2$

<u>Steps</u>	<u>Example</u>	<u>Steps</u>	<u>Example</u>
Parentheses	$+ 2[8 - 4] - (+ 3) + 5^2 - 20 \div 2$	Neighbor-sign rule	$= + 8 - 3 + 25 - 10$
	$= + 2[4] - (+ 3) + 5^2 - 20 \div 2$	Grouping	$= + 8 + 25 - 3 - 10$
Exponents	$= + 2[4] - (+ 3) + 25 - 20 \div 2$	Same-sign rule	$= + (8 + 25) - (3 + 10)$
Multiplication	$= + 8 - (+ 3) + 25 - 20 \div 2$		$= + 33 - 13$
Division	$= + 8 - (+ 3) + 25 - 10$	Mixed-sign rule	$= + (33 - 13)$
			$= + 20$



Now try using the order of operations yourself:

a) $8^2 - (-10) + 25 \div 5$

b) $3[4^2 - (5 + 2)]$

c) $8 - (-3) + (-4) - 3(10 - 7)$

d) $(-2) \cdot (-3) + 6\{(12 \div 6) + 2\}$

e) $7 + 2[(10 - 8)(5 - 2) - 4] - 3^2$

Answers:
a) 79
b) 27
c) 2
d) 30
e) 2